Rethinking Social Action.
Core Values in Practice

Correctness of Syllogistic Reasoning

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Abstract

The syllogistic inferences with cognitive statements are formal structures of thinking researched even from the Greek antiquity and make the subject of logics. The research of the modalities in which ordinary people reason in syllogistic structures is of a much more recent date, since the last century, and make the object of cognitive psychology. Our empirical study aims to investigate the correctness of thinking in various schemes of syllogistic reasoning. For this purpose we applied a set of four questionnaires, comprising 16 syllogisms each, of which 8 with affirmative conclusion and 8 with negative conclusion; 8 modes with universal conclusion, 8 modes with particular conclusion; each questionnaire includes four modes of each syllogistic figure. The first questionnaire contains complete syllogisms in formal expression, which the respondent must evaluate in terms of correctness. The second questionnaire contains the same premises as the first one, but the subjects are required to draw the proper conclusion themselves. Questionnaires 3 and 4 are analogous to the first two, except that this time syllogisms are formulated in natural language. Statistical processing involved comparing the number of correct answers in relation to the variables studied – syllogistic figure, valid/invalid syllogistic mode, affirmative/negative, universal/particular conclusion. The research was replied having been obtained similar results.

Keywords: syllogism; syllogistic modes; syllogistic figures; cognitive psychology.

1. Introduction

Syllogism is the most frequently used type of reasoning in arguments and demonstrations, in discourses of scientific or ordinary nature. Logically,
the simple categorical syllogism is the inference that leads from two
categorical sentences with a common term (also called middle term - M) to the
deduction of another categorical sentence containing the non-common
terms (also called extreme, S and P) of the first two.

For example:

No mammal is invertebrate
All does are mammals
No doe is invertebrate

Component sentences vary by quantity (the fact of being universal or
particularly quantified) and by their quality (the fact of being affirmative or
negative). Combining criteria, there are four types of categorical sentences:
universally affirmative (All A are B, symbolized by "a"), universally negative
(No A is B, symbolized by "e"), particularly affirmative (Some A are B,
symbolized "i") and particularly affirmative (Some A is not B, encoded with
"o"). Using these symbols, the syllogistic mode given as an example may be
shortened as eae-I, with the last symbol in the abbreviation corresponding to
the syllogistic figure.

According to the position of the common term in the structure of
premises, there are four possible combinations, called syllogistic figures. In our
study we used scholastic numbering of syllogistic figures, well established in
studies of logic, contrary to other researchers of this issue [3], [10], namely:

<table>
<thead>
<tr>
<th>Figure I</th>
<th>Figure II</th>
<th>Figure III</th>
<th>Figure IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>M-P</td>
<td>P-M</td>
<td>M-P</td>
<td>P-M</td>
</tr>
<tr>
<td>S-M</td>
<td>S-M</td>
<td>M-S</td>
<td>M-S</td>
</tr>
<tr>
<td>S-P</td>
<td>S-P</td>
<td>S-P</td>
<td>S-P</td>
</tr>
</tbody>
</table>

By combining the 4 sentences taken 3, we can identify 64 possible
combinations, called syllogistic modes, for each figure. Of the 256 possible
syllogistic modes, only 19 are valid, i.e. from the stated premises stated there
follows with necessity the logical conclusion. Considering also the weak
(subordinate) modes, there are still 24 modes that could be considered
correct, 6 for each figure. In our research, we accepted as valid the
subordinate modes too (those that derived a particular conclusion although a
universal conclusion could have been generated, starting from the premise
that if a universal sentence is true, then its subordinate particular is also
ture). We also considered valid those modest hat derived a conclusion by
legal conversion of a negative universal or an affirmative particular,
reasoning that such sentences are equivalent from a logical point of view
(which strictly formally involves changing the syllogistic mode from one
figure to another); considering also the possibility of conclusion conversion, the number of possible modes is doubled, getting to 512, as performed by Khemlani & Johnson-Laird [10].

2. Problem Statement

Given this structural simplicity and the finite and relatively low number of valid combinations, the syllogism was “not only the first form of reasoning researched and theorized on, but also the most widely studied over the centuries” [5]. In psychology, categorical syllogism has been studied for more than a century, starting with Störring, 1908 [3]. The means of psychological research of the syllogistic reasoning are very diverse: the subjects under scrutiny may be asked to qualify as correct or false a complete syllogistic argument; only premises are presented and the subjects are required to find the resulting conclusion by themselves, or to select the resulting conclusion from a list; syllogism may be given in natural or simplified language, in formal language, using symbols (A, B, C); the subjects may also be required to verbalize their way of reasoning or they may be video-recorded to see how they react, what thinking schemes they are using on the answering draft; they may or may not be timed etc. After such experimental studies, a number of theories have been formulated, trying to explain how are people thinking when they reason syllogistically. Some theories explain reasoning processes by using certain rules of mental logic, or inference rules [2]; [12], and these are called syntactic or sentential theories [7], while others explain syllogistic reasoning by using certain mental models [9], and these are called semantic theories. Naturally, there are also some synthetic theories which aim to overcome the semantic-syntactic dichotomies, and these are called of dual processes or mechanisms [6]. In order to identify sources of fallacies in syllogistic argumentation, heuristic theories invoke the atmosphere effects [15] or the “context effect” [2], the erroneous interpretation of premises or of the relation between terms [4]; [11]; [1], particularities of how quantifiers are interpreted [13]), limits of working memory etc. In a recent article, Khemlani & Johnson-Laird, [10] consider that there are 12 theories in cognitive psychology about syllogistic reasoning, qualifying the situation as “a small disaster” of any scientific field and they suggest some reference points for a unified theory.
3. Research Questions

Our research was not intended to formulate a competing theory to those already existing in cognitive psychology, nor to empirically validate any one of them. Our goals are much more modest, and they have been set out of epistemic curiosity and based mainly on educational imperatives, concerned firstly with teaching the discipline of Logic. Research queries: is there any difference, significant statistically, regarding the efficiency of reasoning, between the four syllogistic figures? This query is based on the fact that Aristotle believed, in his Prior Analytics, that figure I was perfect from a logical point of view. Does this also result in the fact that thinking is more efficient in its natural exercise in the modes of this figure? Is there a statistically significant difference from the standpoint of thinking efficiency (of its validity or correctness) between a reasoning expressed in formal language and that expressed in natural language? How about between evaluating a complete categorical syllogism as being valid or invalid and deriving a correct conclusion from the premises presented? In other words, between the simple categorical syllogism and 3rd order enthymeme? Which are the additional factors which can influence the correctness of reasoning? Syllogistic mode, namely the quality or quantity of premises? Familiarity of the terms used or the empirical obviousness of the conclusion? The empirically-based answers to such queries are likely to start a field of cognitive psychology research for students of psychology colleges enrolled in an optional course of logic.

4. Research Methods

To answer the above mentioned queries, subjects have been handed four sets of questionnaires. The first questionnaire presents 16 complete syllogisms, expressed in formal language, and the subjects are asked to evaluate the syllogism as being correct or incorrect. For example:

- No A is B
- All C are A
- No C is A

The questionnaire was so conceived to contain four modes for each syllogistic figure, of which two affirmative modes and two negative modes, two with universal conclusions and two with particular conclusions. Regarding their validity, 8 of the proposed syllogisms have been valid and the other 8 invalid, and the questionnaires were conceived reflecting figures,
so three correct ones for figure I, three incorrect ones for those in figure IV; two correct and two incorrect ones for figures two and three. To clarify the explanation, the table below indicates the validity of syllogisms used, noting conventionally validity with 1 and invalidity with 0:

<table>
<thead>
<tr>
<th>Figure I</th>
<th>Figure II</th>
<th>Figure III</th>
<th>Figure IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

We aimed that strictly formally, all variables used (figure, affirmative, negative, universal, particular, valid, invalid modes) be homogeneous and symmetrically arranged related to the figure. The second questionnaire is identical to the first one, except that this time the conclusion is not indicated, rather the students are asked to derive by themselves the conclusion they think would follow from the premises stated, and if there is no conclusion, they should mention this fact. We used this procedure to see whether there is any significant difference in terms of reasoning correctness between the two tests, of which the first one requires a mere opinion and, therefore, labeling (correct/incorrect), in which case there is a high risk of a “random” answer, while the second one requires actual thinking. We considered the comment of A. Surdu [14], stating that in case of presenting a complete reasoning, “no syllogistic thinking is performed, but rather syllogism itself becomes the object of reasoning, as it is given as such” [14, 15] and the idea that a correct answer is not necessarily the result of a correct reasoning [8].

While the first two questionnaires have been expressed in formal language, using symbols (A, B, C), the next two questionnaires contain syllogisms expressed in natural language, using the structure of the first two, following the scheme of syllogistic figures, i.e. questionnaire I mirrors questionnaire IV and questionnaire II mirrors questionnaire III. In other words, in questionnaires three and four all we did was add matter (natural language) to the forms of the first two questionnaires, aiming at a balance of familiarity between the terms used.

Considering the goal we had, the questionnaires were presented without imposing a time limit and without timing the length of answers or video-recording the subjects’ behavior or their methods of solving. The syllogistic sentences have been expressed with standard quantifications
Participation was voluntary and anonymous, but subject to an approval to processing data strictly for scientific purposes.

The subjects of the research were students of Psychology and Social Work specializations at the "Ovidius" University of Constanta, most of them completing at least three years ago a discipline in which they also studied in a basic form syllogistic reasoning. Some questionnaires (incomplete or suspected to be answered without implication) were eliminated. The first questionnaire was fully answered by 74 subjects, the second questionnaire was answered by 40 subjects, the third by 57 subjects and the last one by 46 subjects.

5. Findings and Discussions

The data collected was processed in SPSS version 20. The percentage of correct answers for each item and questionnaire are presented in the following table:

Table no. 3 Percentage of correct answers for each item

<table>
<thead>
<tr>
<th>Fig.</th>
<th>I1</th>
<th>I2</th>
<th>I3</th>
<th>I4</th>
<th>I5</th>
<th>I6</th>
<th>I7</th>
<th>I8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q1</td>
<td>91.89</td>
<td>93.24</td>
<td>27.03</td>
<td>77.03</td>
<td>39.19</td>
<td>70.27</td>
<td>48.65</td>
<td>51.35</td>
</tr>
<tr>
<td>Q2</td>
<td>80</td>
<td>70</td>
<td>20</td>
<td>5</td>
<td>12.5</td>
<td>92.5</td>
<td>17.5</td>
<td>12.5</td>
</tr>
<tr>
<td>Q3</td>
<td>56.14</td>
<td>54.39</td>
<td>1.75</td>
<td>0</td>
<td>82.46</td>
<td>29.82</td>
<td>14.04</td>
<td>64.91</td>
</tr>
<tr>
<td>Q4</td>
<td>95.65</td>
<td>95.65</td>
<td>21.74</td>
<td>76.26</td>
<td>89.13</td>
<td>63.04</td>
<td>54.35</td>
<td>50</td>
</tr>
<tr>
<td>Total</td>
<td>80.92</td>
<td>78.32</td>
<td>17.63</td>
<td>39.57</td>
<td>55.82</td>
<td>63.90</td>
<td>33.63</td>
<td>44.69</td>
</tr>
</tbody>
</table>

Analyzing answers for each item it results that the most correct answers (95.65%) belong to the first two items ("warm up"), the first item being the syllogism exemplified in the introduction and the second one is as follows:

*No feline is biped*

*All penguins are biped*

*No penguin is a feline*
Familiarity with the terms used and the empirical evidence of the conclusion have favoured labelling as valid syllogism. By contrast, there is a syllogistic mode that has received no correct answers, i.e. item 4 of questionnaire 3. The task was to derive the conclusion of the following premises: *No athlete is sedentary / All sedentary people are easy-going.* Most respondents expressed the conclusion: *No easy-going person is an athlete* or *No athlete is easy-going.* There is an atmosphere effect here, where the universality of premises suggests the universality of conclusion. One expression: *No B is A / All A’s are C’s,* where most answers for those who draw a conclusion were like: *No C is B or No B is C.* Interestingly, when the conclusion *No C is B* is present and the syllogism qualification as correct or incorrect is required, the majority of responses (77.03%) intuited that this conclusion does not follow. Likewise, when presenting the syllogism in natural language, 76.26% of the answers were correct (“syllogism is not valid”, “the conclusion does not follow”).

A closely related difficulty is encountered by this kind of reasoning whatever the figure, only 1.75% of the answers were correct both in Figure III and Figure II (items 3 and 11) of questionnaire 3. Therefore, the conclusion of other studies regarding the difficulty of mode EAO regardless of syllogistic figure is confirmed, because the conclusion in this case does not follow as an effect of transitivity.

Adding the percentage of correct answers on the four questionnaires, it follows the following table:

<table>
<thead>
<tr>
<th></th>
<th>Fig. I</th>
<th>Fig. II</th>
<th>Fig. III</th>
<th>Fig. IV</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q1</td>
<td>55,77%</td>
<td>68,91%</td>
<td>44,25%</td>
<td>60,13%</td>
<td>57,26%</td>
</tr>
<tr>
<td>Q2</td>
<td>66,25%</td>
<td>80,62%</td>
<td>45,62%</td>
<td>41,25%</td>
<td>58,43%</td>
</tr>
<tr>
<td>Q3</td>
<td>42,98%</td>
<td>37,28%</td>
<td>17,1%</td>
<td>37,28%</td>
<td>33,66%</td>
</tr>
<tr>
<td>Q4</td>
<td>72,82%</td>
<td>61,95%</td>
<td>55,97%</td>
<td>51,63%</td>
<td>60,59%</td>
</tr>
<tr>
<td>Total</td>
<td>59,45%</td>
<td>62,19%</td>
<td>40,73%</td>
<td>47,57%</td>
<td>52,48%</td>
</tr>
</tbody>
</table>

It is noted that questionnaire 4 which shows the fully formulated syllogism in natural language and requires assessment as correct or incorrect, registers the highest rate of correct answers, and questionnaire 3, which requests deriving a conclusion from premises expressed in natural language, has the lowest rate of correct answers. To see if the differences are statistically significant we have operated with the multiple comparison test:
Table Nr. 5. *Multiple Comparisons*

<table>
<thead>
<tr>
<th>(J) Testing</th>
<th>Mean Difference (I-J)</th>
<th>Std. Error</th>
<th>Sig.</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-1.107</td>
<td>.392</td>
<td>.993</td>
<td>-1.12 - 0.91</td>
</tr>
<tr>
<td>2</td>
<td>3.857</td>
<td>.352</td>
<td>.000</td>
<td>2.94 - 4.77</td>
</tr>
<tr>
<td>3</td>
<td>-.452</td>
<td>.376</td>
<td>.624</td>
<td>-1.42 - 0.52</td>
</tr>
<tr>
<td>4</td>
<td>1.107</td>
<td>.392</td>
<td>.993</td>
<td>-0.91 - 1.12</td>
</tr>
<tr>
<td>2</td>
<td>3.964</td>
<td>.413</td>
<td>.000</td>
<td>2.90 - 5.03</td>
</tr>
<tr>
<td>3</td>
<td>-.346</td>
<td>.432</td>
<td>.855</td>
<td>-1.47 - 0.77</td>
</tr>
<tr>
<td>4</td>
<td>3.857</td>
<td>.352</td>
<td>.000</td>
<td>-4.77 - 2.94</td>
</tr>
<tr>
<td>3</td>
<td>4.310</td>
<td>.413</td>
<td>.000</td>
<td>-5.03 - 2.90</td>
</tr>
<tr>
<td>2</td>
<td>-.431</td>
<td>.396</td>
<td>.000</td>
<td>-5.34 - 3.28</td>
</tr>
<tr>
<td>1</td>
<td>.452</td>
<td>.376</td>
<td>.624</td>
<td>-0.52 - 1.42</td>
</tr>
<tr>
<td>4</td>
<td>.346</td>
<td>.432</td>
<td>.855</td>
<td>-0.77 - 1.47</td>
</tr>
<tr>
<td>3</td>
<td>4.310</td>
<td>.396</td>
<td>.000</td>
<td>3.28 - 5.34</td>
</tr>
</tbody>
</table>

* The mean difference is significant at the 0.05 level.

Significant differences between questionnaires 1 and 3, 2 and 3, 3 and 4 to p = 0.000 are noted from the table, but not between the others. This fact reasons the idea that deriving a conclusion from the premises stated in natural language is much more difficult than deriving a conclusion in a formal language or than assessing the accuracy of a syllogistic reasoning regardless of its form of expression, in formal or natural language.

Comparing the correct answers from questionnaires I and II (in formal language) aggregated (percentage mean of 57.84) with questionnaires III and IV (in natural language) (percentage mean of 47.12), it results a significant difference in test Z = 10.29, > 2.58 to p <0.01. This indicates that, at least at students’ level, operating in formal language seemed to be easier than operating with natural language reasoning. Such a situation justifies both the theories of inference rules and the theory of mental models, because both theories assume implicitly or explicitly (using Euler type graphic representations) a symbolic type operation by translating natural language in circles that relate terms or in formal expression rules.

Analyzing the correctness of answers on each of the syllogistic figures, we find on the total of questionnaires, that the highest percentage of correct answers is recorded in Figure II (62.19%) and the lowest in Figure III (40.73%). Applying the Z test for comparing two independent samples...
to figures II and III there is a significant difference in the rate of correct answers, \( Z = 7.81, > 2.56 \) at a lower materiality threshold of 0.01. The highest distance between the two figures is found in questionnaire 2, where the rate of correct answers for Figure II is 80.62%, while for Figure III is only 45.62%. It would seem that from a logical point of view, Figure I is a perfect figure while from a psychological point of view, the "perfect" figure is the second one. However, there are no significant differences between Figures I and II (\( Z = 1.25, <1.96, p > 0.05 \)). If we operate on the formal/material distinction, we find that if we formulate reasoning in natural language, Figure I has the highest percentage of correct answers (57.9%), Figure II hovering in second place (49.61%). In both cases, Figure III has the lowest percentage.

In terms of quality of syllogistic mode (affirmative or negative conclusion), the differences on all tests are not significant, but become significant in some questionnaires and figures. The same is true in terms of quantity (universal/particular conclusion). However, the greatest difficulties are encountered in case of modes with both universal premises deriving a particular conclusion, the limit case being represented by EAO mode, especially in Figure III.

6. Conclusions

Summarizing the findings, we can say that:
- the accuracy of reasoning depends mainly on the content of premises not on the syllogistic figure;
- Figures II and II are the easiest for reasoning;
- Figure III is the most difficult for reasoning;
- the most difficult mode (among those researched) seems to be eao-III, but it remains difficult for the other figures too;
- the easiest modes seem to be eae-I and eae-II;
- the most difficult modes are those deriving a particular conclusion from two universal premises;
- when we need to derive a conclusion, it is easier if premises are expressed in formal, rather than natural, language;
- it is easier to evaluate the correctness of a syllogistic mode if it is expressed in natural language.

Considering the limits of our research, its reduced sample group, the low number of analyzed syllogisms etc., some assertions must be taken cautiously until their empirical reconfirmation. Although our research can neither confirm nor invalidate a certain psychological theory on syllogistic
reasoning, the whole data we collected makes us rather favor a synthetic
type, which partially confirms heuristic theories of inference rules and
mental models. To decide between these theories (from a Popperian
perspective), a crucial experiment would be necessary, with a potential
falsifier, yet to be identified.

References

[2] Braine, M. D. S, Rumain, B. Logical reasoning. In: Editor Flavell J. H,
Editor Markman E. M. Carmichael`s handbook of child psychology, vol
of Experimental Psychology. 1959; 58 (3): 220.
[Classical syllogistics and modern interpretations]. Bucharest: Didactic and
[6] Faicuț, L. E. Recent theories on the processes of syllogistic reasoning and
their explanatory limits. History Institute „G. Bariț” of Cluj-Napoca, Series
[8] Iancu, L. Locul silogismului categoric in studiile de psihologie a
raționamentului [Position of Categorical Syllogism in Studies of Psychology
of Reason]. ANNALS of the University of Bucharest, Philosophy, Series.
reasoning: Rational decision from personalized representations. In: Editor
1978.
[13] Roberts, M. J., Newstead, S. E., Griggs, R. A. Quantifier Interpretation and
[14] Surdu, A. Semnificația psihologică a silogismului categoric [Psychological
Meaning of Categorical Syllogism]. In Cercetări filosofico-psihologice